Cosmology Notes

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1 Introduction to Modern Cosmology

Cosmology is the scientific study of the large-scale properties of the universe as a whole. It endeavors to use the scientific method to understand the origin, evolution, and ultimate fate of the entire universe. For centuries, cosmology was primarily a philosophical or theological discipline. It was not until the early 20th century, with the advent of Albert Einstein's Theory of General Relativity, that cosmology truly became a physical science.

The fundamental premise of modern cosmology is that the physical laws we observe on Earth and in our solar system apply everywhere in the universe. This assumption allows us to interpret astronomical observations of distant galaxies and the cosmic microwave background radiation to reconstruct the history of the cosmos. In this extensive document, we will follow the narrative thread of fundamental astronomical observations and trace how they lead directly to the Standard Cosmological Model (ΛCDM), exploring the theories of gravity, the dynamics of expansion, the formation of cosmic structures, and the profound mysteries of dark matter and dark energy.

2 Fundamental Cosmological Observations

Modern cosmology is built upon three pillars of observational evidence: the expansion of the universe, the cosmic microwave background radiation, and the primordial abundance of light elements.

2.1 The Doppler Effect, Galaxy Recession, and Hubble's Law

When observing the spectral lines of distant galaxies, astronomers noticed that the characteristic emission and absorption lines of known elements were shifted from their laboratory wavelengths. This phenomenon is a manifestation of the Doppler effect.

If a light source is moving relative to an observer, the observed frequency f is related to the emitted frequency f_0 by:

$$f = f_0 \frac{v}{v + v_r} \tag{1}$$

where v is the wave speed and v_r is the radial velocity of the source. For electromagnetic waves, we often express this in terms of the wavelength shift:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c} \tag{2}$$

where c is the speed of light.

- **Blueshift**: If the source is approaching, the wavelength decreases (variation is negative), shifting the light toward the blue end of the spectrum.
- **Redshift**: If the source is moving away, the wavelength increases (variation is positive), shifting the light toward the red end of the spectrum.

In 1929, Edwin Hubble made a groundbreaking discovery [6]: the vast majority of galaxies exhibit a redshift, meaning they are moving away from us. Furthermore, he found that the recession velocity v_r of a galaxy is directly proportional to its distance d from Earth. This relationship is codified as **Hubble's Law**:

$$v_r = H_0 \cdot d \tag{3}$$

where H_0 is the Hubble constant. It is essential to understand that H_0 is constant in space at a given epoch, but it varies over cosmic time.

A critical question arises: Does this imply that Earth is at the center of the universe? The answer is no. The recession of galaxies is an optical effect of the *expansion of space itself*. Just as dots drawn on a balloon move apart as the balloon is inflated, galaxies move apart as the fabric of the universe stretches. Every observer in the universe would see the same generalized expansion.

2.2 The Cosmic Microwave Background Radiation (CMB)

In 1965, Arno Penzias and Robert Wilson inadvertently discovered a persistent, isotropic microwave noise while calibrating a highly sensitive radio antenna [8]. This signal had the same intensity in all directions and did not originate from the Earth, the Sun, or our galaxy. They had discovered the Cosmic Microwave Background (CMB).

The CMB is perfectly described by a black body energy spectrum with a temperature of approximately 2.726 K. A black body spectrum is defined by Planck's law:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \left(\exp \frac{h\nu}{k_B T} - 1 \right)^{-1} \tag{4}$$

In terms of wavelength:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \left(\exp \frac{hc}{\lambda k_B T} - 1 \right)^{-1} \tag{5}$$

For radiation to possess a perfect black body spectrum, it must have been in thermal equilibrium with matter at some point in the past. This requires matter to have been in a highly ionized state (a plasma) so that photons and electrons could frequently interact via Thomson scattering. The interaction rate is given by $I = n_e \cdot \sigma_T \cdot c$, where n_e is the electron number density and $\sigma_T = 6.6 \times 10^{-25}\,\mathrm{cm}^2$ is the Thomson cross-section.

In the present universe, the mean time interval between collisions is vast ($\tau \sim 8 \times 10^{12}$ years), meaning matter and radiation are completely decoupled. The existence of the CMB thus proves that the early universe was incredibly dense and hot, allowing radiation and matter to be tightly coupled. The extreme isotropy of the CMB (temperature variations $\Delta T/T \sim 10^{-5}$) also provides profound evidence that the very early universe was remarkably homogeneous and isotropic.

2.3 The Abundance of Light Elements

The final major observational pillar is the abundance of light elements, specifically Hydrogen, Helium, Deuterium, and Lithium. In stars, nuclear fusion synthesizes heavier elements from lighter ones. However, the observed abundance of Helium (about 25% by mass) in the universe is far too high to have been produced entirely by stellar nucleosynthesis over the universe's lifetime. Moreover, elements like Deuterium are actually destroyed in stellar cores, not created.

This implies that these light elements must have been synthesized outside of stars, during an epoch when the entire universe was hot and dense enough to sustain nuclear fusion. This period, known as Primordial Nucleosynthesis, occurred within the first three minutes after the Big Bang.

Astronomers measure the primordial abundance of these elements by observing the spectra of distant quasars. Quasar light passes through intergalactic gas clouds on its way to Earth. Photons with specific energies are absorbed, creating absorption lines such as the Lyman-α forest (associated with the $n = 1$ to $n = 2$ transition of neutral hydrogen at $\lambda = 121.567$ nm). The precise characteristics of these lines allow us to deduce the chemical composition of the primordial gas clouds, beautifully confirming the predictions of Big Bang nucleosynthesis [1].

3 The Cosmological Principle

Synthesizing these observations leads us to the foundation of cosmological models: **The Cosmological Principle**, formally articulated by George Gamow in 1948 [4].

The Cosmological Principle states that, on sufficiently large scales, the universe is:

1. **Homogeneous**: It has the same properties at every point.
2. **Isotropic**: It looks the same in every direction.

While the present-day universe is clearly highly anisotropic and inhomogeneous on smaller scales, organized into stars, galaxies, clusters, and a vast "cosmic web", averaging over scales larger than about 100 Megaparsecs restores homogeneity. Furthermore, the extreme smoothness of the CMB demonstrates that the early universe was nearly perfectly homogeneous and isotropic. This principle simplifies the complex equations of gravity, allowing us to describe the dynamics of the entire universe mathematically.

4 The Theories of Gravity

Because cosmology deals with the largest scales imaginable, the only relevant fundamental force is gravity. Electromagnetic forces tend to cancel out due to the neutrality of macroscopic matter, and nuclear forces operate only on subatomic scales.

4.1 From Newton to Poisson

Isaac Newton formulated the universal law of gravitation, stating that the force between two masses is:

$$\vec{F} = GMm\frac{\vec{x'} - \vec{x}}{|\vec{x'} - \vec{x}|^3} \tag{6}$$

For a continuous mass distribution ρ, the gravitational force on a unit mass can be expressed as the negative gradient of a gravitational potential Φ:

$$\vec{F}(\vec{x}) = -\vec{\nabla}\Phi \tag{7}$$

This leads to the **Poisson Equation**, which relates the potential to the density of matter:

$$\vec{\nabla}^2\Phi(\vec{x}) = 4\pi G\rho(\vec{x'}) \tag{8}$$

In Newtonian gravity, matter dictates the gravitational field. However, Newtonian gravity assumes absolute space and time and implies instantaneous action at a distance, concepts incompatible with the speed of light limit.

4.2 Special and General Relativity

Special Relativity introduced the equivalence of mass and energy ($E = \gamma mc^2$), meaning that energy itself possesses inertia and acts as a gravitational source. General Relativity (GR), published by Einstein in 1915 [3], revolutionized gravity. Instead of a force propagating through a static background, gravity in GR is the curvature of space-time itself.

The relationship between the geometry of space-time and the distribution of mass-energy is encapsulated in the **Einstein Field Equations**:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{9}$$

Here, $G_{\mu\nu}$ is the Einstein tensor, which describes the curvature and geometry of space-time. $T_{\mu\nu}$ is the energy-momentum tensor, which describes the distribution of energy, momentum, and pressure. Succinctly: matter tells space-time how to curve, and space-time tells matter how to move.

5 The Cosmological Model

5.1 The Space-Time Metric and FRW Geometry

To solve Einstein's complex non-linear equations for the entire universe, we apply the Cosmological Principle. Homogeneity and isotropy dramatically simplify the geometry of

space-time, leading to the **Friedmann-Robertson-Walker (FRW) metric**:

$$ds^2 = c^2dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right] \tag{10}$$

This equation calculates the infinitesimal distance ds between two events.

- $a(t)$ is the **scale factor**, an expanding (or contracting) parameter that dictates how the physical distance between co-moving coordinates changes over time.
- k represents the **curvature** of space. Due to homogeneity, k must be constant everywhere.

5.2 The Curvature of the Universe

The geometry of the universe is classified into three distinct categories based on the value of k and the mass-energy density ρ:

1. **Open Universe** ($k < 0, \rho < \rho_c, \Omega < 1$): The geometry is hyperbolic (like a horse's saddle). Parallel lines diverge, and the sum of triangle angles is less than 180°. The universe expands infinitely with a non-zero asymptotic velocity.
2. **Flat Universe** ($k = 0, \rho = \rho_c, \Omega = 1$): The geometry is Euclidean (flat). Parallel lines remain parallel, and triangle angles sum to 180°. The universe expands forever, but the expansion velocity approaches zero asymptotically.
3. **Closed Universe** ($k > 0, \rho > \rho_c, \Omega > 1$): The geometry is spherical. Parallel lines converge, and triangle angles sum to more than 180°. The universe expands to a maximum size and then contracts in a "Big Crunch."

The critical density ρ_c, defining the threshold between these geometries, is given by $\rho_c = \frac{3H^2}{8\pi G}$. The density parameter is defined as $\Omega = \frac{\rho}{\rho_c}$. Current observations suggest our universe is extremely close to flat ($\Omega \approx 1$).

5.3 The Friedmann Equations

By inserting the FRW metric and an ideal fluid energy-momentum tensor into Einstein's equations, we obtain the **Friedmann equations**, which govern the expansion dynamics of the universe [12]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{11}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{12}$$

The first equation links the expansion rate (the Hubble parameter $H = \dot{a}/a$) to the density and curvature. The second equation represents the acceleration of the expansion. Notice

that for standard matter and radiation (where density ρ and pressure P are positive), the term on the right is negative, implying that gravity acts to decelerate the expansion ($\ddot{a} < 0$).

6 History of Universe Expansion and Cosmic Structures

6.1 Thermal History: Radiation, Matter, and Equivalence

The history of the universe is defined by how the densities of its various components scale with expansion.

- **Matter**: The density of non-relativistic matter scales as $\rho_m \propto a^{-3}$ due to simple volume expansion.
- **Radiation**: The density of radiation scales as $\rho_r \propto a^{-4}$. This steeper decline is because, in addition to volume expansion, the wavelength of each photon is stretched by the expanding universe, draining its energy.

Because radiation density falls faster than matter density, the early universe was **Radiation-Dominated**. As it expanded and cooled, it reached the **Epoch of Equivalence** (at redshift $z \approx 1600$), where matter and radiation densities were equal. Subsequently, the universe became **Matter-Dominated**.

Around $z \approx 1100$, the temperature dropped low enough (about 4000 K) for protons to capture electrons, forming neutral hydrogen. This is the **Epoch of Recombination**. Shortly after, at $z \approx 300$, the absence of free electrons drastically reduced the rate of Thomson scattering. Photons decoupled from matter and began streaming freely across the universe. This **Epoch of Last Scattering** generated the Cosmic Microwave Background we observe today.

6.2 The Formation of Cosmic Structures

Despite the overall homogeneity described by the Cosmological Principle, the universe contains complex structures. How did these form? The standard paradigm is **Gravitational Instability**. Tiny primordial density fluctuations present in the early universe (which we observe as tiny temperature variations in the CMB) were amplified over billions of years by gravity.

There are two main hypothetical scenarios for structure formation:

- **Top-Down Scenario**: Huge supercluster-sized structures form first and then fragment into individual galaxies.

- **Bottom-Up (Hierarchical) Scenario**: Small, sub-galactic halos form first. Over time, these merge and accrete to form galaxies, which then group into clusters and superclusters.

Observational evidence heavily favors the **Bottom-Up** scenario. Advanced N-body numerical simulations [13] show that small primordial density perturbations naturally evolve into the intricate "Cosmic Web" of filaments, nodes (galaxy clusters), and vast voids we observe today.

7 Problems and Solutions in the Standard Model

While immensely successful, the Standard Cosmological Model faces critical challenges that have led to paradigm-shifting theoretical extensions.

7.1 The Horizon and Flatness Problems

- **Horizon Problem**: The CMB is incredibly uniform across the entire sky. However, points on opposite sides of the sky were separated by distances greater than light could have traveled between the Big Bang and the Epoch of Last Scattering. How could these causally disconnected regions reach perfect thermal equilibrium?
- **Flatness Problem**: The universe is observed to be highly flat today ($\Omega \approx 1$). However, any deviation from $\Omega = 1$ in the early universe would have been amplified drastically over time. For the universe to be flat today, it must have been flat to an astonishing degree of precision in the primordial epoch (a massive fine-tuning problem).

Solution: Cosmic Inflation. In the 1980s, physicists [5]proposed that between 10^{-36} and 10^{-32} seconds after the Big Bang, the universe underwent an exponential, violently accelerated expansion. Inflation stretches the universe so rapidly that it drives any initial curvature to zero (solving the Flatness Problem). It also means that the entire observable universe originated from a tiny, causally connected patch that was in thermal equilibrium before inflation began (solving the Horizon Problem). Furthermore, quantum fluctuations during inflation provide the seeds for the density perturbations that eventually formed galaxies.

7.2 Dark Matter

Observations of galactic rotation curves present a profound anomaly. According to Newtonian dynamics and the observable distribution of stars and gas, the rotational velocity of galaxies should decrease at large radii (Keplerian decline). Instead, observations by

Vera Rubin [11]and others showed that rotation curves remain flat far beyond the visible galactic disk.

This implies the presence of a vast, invisible halo of mass surrounding galaxies. Similar anomalies are found when calculating the virial mass of galaxy clusters and observing gravitational lensing. This mysterious substance, which does not interact with electromagnetism but exerts a powerful gravitational pull, is known as **Dark Matter**. It constitutes roughly 27% of the universe's total energy budget and is crucial for the formation of cosmic structures in the hierarchical model.

7.3 Dark Energy

In the late 1990s, observations of distant Type Ia supernovae, which serve as "standard candles" for measuring cosmic distances, revealed that the expansion of the universe is not decelerating due to gravity, but is actually **accelerating** [10, 9].

Looking back at the second Friedmann equation, for acceleration ($\ddot{a}$) to be positive, there must exist a component of the universe with a strongly negative pressure ($P < -\rho c^2/3$). This exotic component is termed **Dark Energy**. The simplest theoretical explanation for dark energy is the Cosmological Constant (Λ), a vacuum energy intrinsic to space itself. Dark energy currently dominates the universe, comprising approximately 68% of its energy budget [12, 7].

8 Conclusion

The modern standard model of cosmology, the ΛCDM model (Lambda Cold Dark Matter), successfully synthesizes a century of observational and theoretical breakthroughs. It describes a universe born in a hot, dense Big Bang, smoothed by an initial burst of inflation, shaped by the gravitational scaffolding of dark matter, and currently being driven apart by the relentless push of dark energy. While dark matter and dark energy remain the most profound mysteries in fundamental physics today, cosmology provides a robust, beautifully mathematically consistent framework for understanding the grandest scales of reality [2].

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